Grade 1 Children in Problem-Posing Contexts: Problem Solving Prior to Completing the Task

Tom Lowrie
Charles Sturt University
<tlowrie@csu.edu.au>

With support from undergraduate university students, Grade 1 children (aged 6-7 years) were challenged to pose and then solve their own open-ended tasks. The way in which the children engaged in problem solving prior to formulating or posing a problem was explored. An overview of the types of problems posed by the class is presented, along with two detailed case studies. The case studies show that the children were able to identify and discuss the type of mathematics content and strategies they would need to employ in order to successfully complete the task.

Problem posing is an important companion to problem solving and lies at the heart of mathematical activity (Kilpatrick, 1987). It has been used to refer both to the generation of new problems and to the reformation of given problems (Silver, 1994). In the first instance, "the goal is not the solution of a given problem but the creation of a new problem from a situation or experience" (Silver, Mammona-Downs, Leung & Kenney, 1996, p. 294).

Silver (1995) identified four types of problem-posing experiences that provide opportunities for children to engage in mathematical activity. He argued that problem posing could occur *prior* to problem solving when problems were being generated from a particular situation, *during* problem solving when the individual intentionally changes the problem's goals or conditions, or *after* solving a problem when experiences from the problem-solving context are modified or applied to new situations. The way in which children engage in problem solving prior to constructing or posing a problem was investigated in the present study.

Constructing Problem-posing Environments

Some researchers (Ellerton, 1986; Mamona-Downs, 1993) have found that, for motivational purposes, it is helpful to have someone in mind when designing problems. In the present study, the friend would be someone in the class below or above that of the problem poser. As well as generating a problem for someone to solve, the Year 1 problem generators would also be asked to articulate why such a problem would be appropriate for the respective problem solver. The feedback obtained from this stage of the process fosters a reflective component of the problem-solving process (Lowrie, 1999; Silver et al, 1996).

Ellerton (1986) found those encouraging students to write problems for a friend was a useful way of understanding that person's mathematical ability. In such problem-solving situations the problem poser is forced to consider the individual for whom they are designing the problem. As Stoyanova (1998) commented:

> ...there is a strong acceptance among researchers and educators of the notion that students' ability in posing quality problems provides a useful indication of potential mathematical talent. (p. 172)

The very fact that a student must consider the mathematical ability of another person when engaged in free problem-solving situations requires reflection and careful planning. In order to complete the task successfully, the problem poser might not only focus on the underlying structures of the problem but also the extent to which the problem solver will be able to interpret the components of the problem. Such metacognitive thinking processes encourage mathematical power. Some educators (eg., Kulm, 1994; Leung, 1996) have argued that learners as assessors complement and even enhance a teacher's understanding of students' mathematical ability. Such assertions are also explored in this paper.

It could also be argued that problem-posing situations allow children to have some control over the curriculum content and the type of learning activities presented in the classroom. Furthermore, the tasks or the activities children construct may provide insights into their beliefs or attitudes they have toward mathematics and the way in which mathematical knowledge is developed.

The Purpose of the Study

The central concern of the study was to investigate the extent to which young children were able to determine the type of mathematical understandings and problem-solving strategies they would need to use in order to solve a problem they had posed. The following research questions were posed:

- Were the children able to identify mathematical activities that may be required to complete the problem?
- Could the children outline the type of problem-solving strategies they would use to solve the task?
- To what extent could the children highlight possible difficulties that may arise when completing the task?

Method

A cohort of fourth-year undergraduate students (n = 25) in their final semester of a Bachelor of Education course were matched, on a one-to-one basis, with Grade 1 children from a school in a large rural city. Each student teacher worked with one of the Grade 1 children for one hour per week for five weeks. During the first two weeks of the study the children were administered a diagnostic assessment task in order to establish a profile of their mathematics ability. The student teachers also gained an understanding of the children's likes and dislikes over this two-week period and developed a strong rapport over this period of time. An awareness of the children's mathematical ability and their preference for investigating particular content areas certainly helped the student-teachers assist the children to pose their own problems.

During the next three weeks of the investigation the children were encouraged to pose their own problems and solve these tasks with the help of the student teacher. It is quite difficult for young children to design appropriate problems without a substantial amount of practice (Ellerton, 1986), specific instruction (Leung, 1996) or guided questioning (Lowrie, 1999). The rationale behind matching a student teacher with each child was to provide support with problem construction. Further, some children required assistance in constructing problems that were challenging but still "solvable" in the given time period. On occasions the student teachers had to use their professional judgement with respect to the degree of input they had in the problem-posing process. It could be argued, however, that the same balancing act occurs in most teaching-learning situations. After the first two sessions the student teachers were required to develop a profile of their Grade 1 child. The profile included information about the child's:

- Performance in space, measurement and number content strands;
- General problem solving ability;
- Affective dimensions of learning (including motivation, task persistence, beliefs about mathematics); and
- General areas of interest.

The profiles were quite detailed and it was apparent that the reflective nature of this activity helped the student teacher establish a comprehensive understanding of the Grade 1 child's mathematical progress.

The Problem-posing Sessions

For each of the three problem-posing sessions, the student teacher was responsible for assisting the Grade 1 child to pose problems that both were open-ended in nature and suitable for the individual's needs. A topic or scenario was negotiated with a problem or series of problems generated from the theme. Once the collaborative team was happy with the task the student teacher attempted to establish whether the Grade 1 child was able to determine the type of understandings and strategies that would be required to complete the task. Importantly, the Grade 1 children knew that they were able to seek assistance from their "teacher" to solve the task. This added another dimension to the study because the children were not inhibited by their inability to complete computations or solve multi-step problems. Thus, the Grade 1 children were challenged to consider the types of strategies and methods they would need to use in order to complete the task without being restricted by a lack of content-specific knowledge.

A series of questions were posed to elicit this information before each child began the task. These questions varied depending on the type of problem posed but included some of the themes presented below.

- What might we collect to build this?
- Where will we get these materials?
- What is going to be the hardest thing to do?
- Will we need to do some mathematics?
- When do you think you will need help from me?
- Where should we begin?

The following section identifies some of the problems posed by the Grade 1 children and investigates in more detail the way in which two children responded to these questions before attempting to solve their problem.

Results

General Observations

Not surprisingly, most of the problems generated initially by the children were quite closed and could be solved within a few minutes. With some guidance, the children began to pose problems that were more open in nature and were generally reflected in a *design and make* or *project* form. In total, nine (36%) of the children posed problems that were solved over a three-week period. Table 1 identifies some of the extended problems posed by the Grade 1 children. It also presents responses from the question *Will we need to do some mathematics?*

The children had posed problems that were diverse in nature and appeared to represent areas of personal interest. Mathematics content involved measurement concepts of time, duration, sequencing, length and mass; space concepts of position, shape and structure; and number concepts of money. Although the problems were open-ended the children were still able to recognise the type of mathematical understandings that where embedded in the problem. This was not to say that they knew fully how to go about solving the problem but that they recognised the type of problems they faced within the context of the entire problem.

Table 1
Types of Problems Posed by the Grade 1 Children and Their Response to the Mathematics Required to Solve the Problem

Description of Problem	Student's Response
Designing the school playground	Measure how big and little things are
	Measure how wide and long things are
	Make sure we put things in the right place
	Count the sides of the sandpit
	Step out how far things are away from other things
Listing chores to be done on a farm	Knowing how long it takes to feed all the animals
	Knowing when to feed the animals
	Writing down the time
Becoming a bank manager for a day	Be able to count the money
	Know where everyone works
	Know what time to open and close the bank
	Know where all the money is kept
Build a playground	Make sure the swings and slippery dip are strong enough
	Make sure everything is the right size
	Have really big trees
	Make sure things are not too close together
Making a board game	Make sure all the squares are the same size
	Have a "miss a turn" or "roll again" in every 12 squares
	Make sure some of the special cards are good and some not good

After the children had highlighted components of the problem that required mathematical thinking, they were encouraged to consider whether some of these activities would be more difficult to complete and to identify where they may need additional help. In some instances the children were able to identify the approaches they would need to employ without too much prompting whereas on other occasions more in-depth probing was required. Generally, the children had "grand" ideas for completing the activity. Some of these approaches were quite ambitious and not practical. In such instances the student teacher encouraged the child to propose another solution. Table 2 provides examples of particular elements of posed problems that the children considered to be difficult to solve and approaches that could be used to complete the task.

It could be argued that these responses were insightful and reflective. Although solutions to these problems would not be obtained for up to three weeks, the children were able to discuss openly approaches they would need to use and in addition identify elements of the problem that may require modifying. Moreover, they were able to identify aspects of the problem that could not be solved without assistance from the student teacher.

Table 2
Components of a Problem Identified as Being Difficult to Solve and Suggested Approaches to Complete Such Activities

Description of Problem	Difficult Elements of Task	Approaches That Could be Used
Designing the school Playground (See Figure 1)	Measuring the height of the big tree	You could stand on a ladder with a very long measure. We could measure the shadow on the ground.
Listing chores to be done on a farm (See Figure 2)	Knowing how long it takes to feed all the animals	We could ask a farmer We could make sure that the more animals we have the longer it takes to feed them.
Becoming a bank manager for a day	Be able to count the money	We could use a calculator but it might be better if you helped me with this
Build a playground	Make sure the swings and slippery dip are strong enough.	We could make the model of you (the teacher) first because you are heavier than me. Then we could make sure you can sit on the swing. If you can I can.
Making a board game	Have a "miss a turn" or "roll again" in every 12 squares.	We could put a cross on each number twelve square and then make sure we have one inside each cross

Analysis of the Students' Responses

Case 1: Jane's playground. Jane decided that she wanted to construct a model of the school playground (See Figure 1). Interestingly, she recognised that it would be too difficult to make a model of the entire playground "because we wouldn't have enough time". The student teacher working with Jane helped her modify the task to include a section of the playground that contained a boundary fence, the sandpit and some large trees.

Jane thought that the best way to begin constructing the model was to position all the important objects on a large piece of paper (which later became a piece of masonite). With respect to positioning these objects, Jane wanted to "step out" how far each landmark was from the other. Not surprisingly, she did not know how to transfer this information accurately onto the paper but did not see this as a problem because "you have to put it where it looks right." Jane did not have an understanding of scale (after all she was in Grade 1) but appreciated that her model needed to represent the 3D world because "that tree will have to be the biggest thing in my model." She also realised that the tree had to be taller than the sandpit was long.

Despite the fact that Jane was not able to develop a scale she felt that it was important to measure everything. Her greatest concern was working out how to measure the large tree. Initially, she thought that her teacher could stand on a ladder with a large tape and she could read the measurement on the ground. Before they had a chance to "act out" her proposal she conceded that the school did not have a ladder that tall. She then hypothesised that they could measure the shadow of the tree in order to ascertain the height of the tree. She was not convinced that this was valid but felt that it must have been a good idea based on the student teacher's reaction.

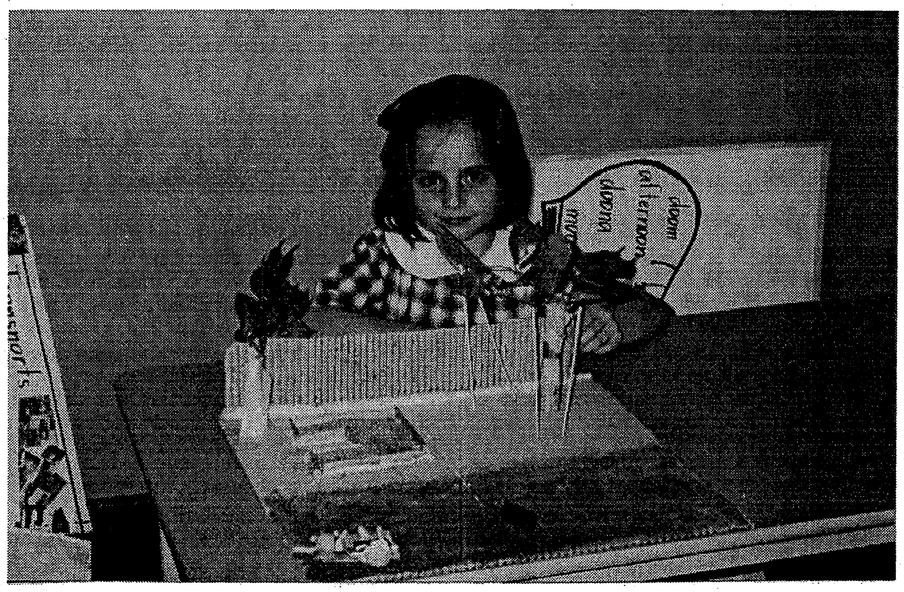

Figure 1. Jane's playground.

Jane was actually creating new problem-posing situations to solve within the context of the original problem she had designed. The open-ended nature of the task provided her with the opportunity to set personal outcomes as the problem evolved.

Case 2: Peter's farm. When Peter originally generated a problem to solve he proposed to build a farm. After discussing the context of the problem with his teacher he modified the task so that it involved describing what chores a farmer and his helper would be required to undertake each day. With some support, Peter recognised that his problem required him to develop timelines of events for the respective characters.

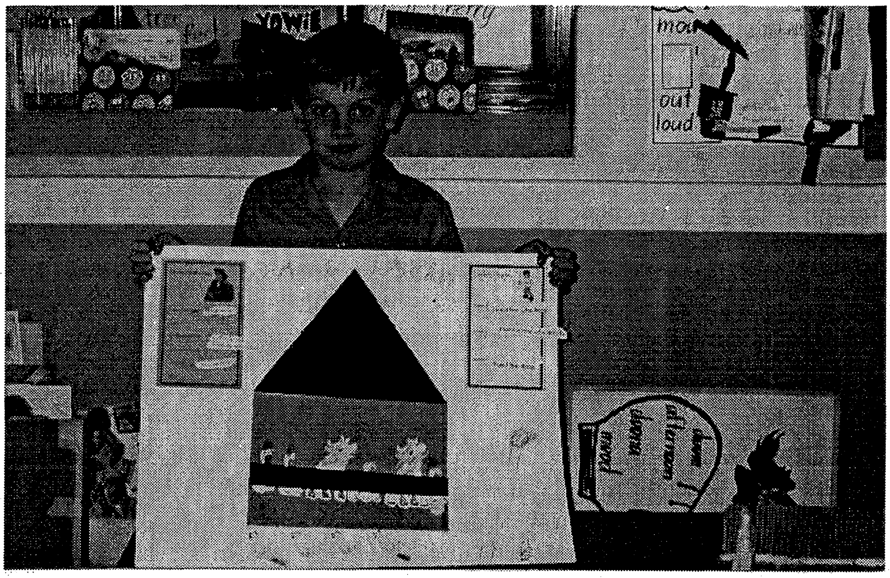

Figure 2. Peter's farm.

Peter decided that he needed to think about what animals were going to be on his farm before he could work out who was going to feed the animals. Thus, Peter demonstrated that he was able to select and monitor relevant problem-solving approaches in a multi-level context. Peter maintained that the most difficult component of his problem was determining the length of time it would take to feed the animals. Importantly, he established a set of criteria that would determine the time it would take to feed the animals with "the more animals we have the longer it takes to feed them." With respect to determining which person would feed particular animals he felt that the farmer should look after all the large animals (eg., cows) while the helper could look after the smaller animals like chickens. Like Jane, Peter was able to develop structures within the open-ended context of the problem that required him to solve additional problems and at the same time established a solution path for the initial problem.

Conclusion

The Grade 1 children described in this study were able to design their own open-ended problems and then collaboratively solve the problem over a three-week period. The children were able to identify mathematical concepts associated with the task and identify areas where they might need assistance. The children were able to consider components of the problem that would be difficult to solve and approaches that could be used to overcome this complexity. Although the children received assistance on a one-to-one basis during the activity, several of the children demonstrated an ability to reason metacognitively.

Several implications emerged from the study.

- Young children are able to generate their own problem-posing contexts if assisted to refine the scope of their investigation.
- Opportunities for engaging in effective problem-solving processes occur prior to children attempting to solve the task.
- Young children are able to modify open-ended tasks if they are assisted to reflect upon the approaches and understandings they require in order to complete the task successfully.

- Some of the most effective open-ended tasks are activities that involve *design and make* or *project* components.

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